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The Effects of a Geometric Redefinition of the Classical Road and Landing Spacing Model Through Shifting

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ABSTRACT. This paper presents a geometric redefinition of the classical road and landing spacing model developed by Matthews (1942), based upon shifting the landings on alternate roads one-half the landing spacing. Through shifting, the average skidding distance is decreased by as much as 8.8 percent. FOREST SCI. 29:670–674.

ADDITIONAL KEY WORDS. Logging, timber production, forest engineering.

THE CLASSICAL LOGGING ROAD AND LANDING SPACING MODEL involves determining the pair of spacings that minimize the total variable cost of timber extraction. The problem was first considered by Matthews (1942), reinforced by Suddarth and Herrick (1964), and later independently unified by Corcoran (1973) and Peters (1978). These efforts were based on the configuration of roads and landings illustrated in Figure 1, where the rectangular areas served per landing have the dimensions of S (road spacing) and W (landing spacing), and represent all the points closest to the landing in question.

Calculation of the theoretical *average skidding distance* (ASD) associated with Figure 1 is an important factor in the determination of the optimum spacings. A mathematically sound ASD formulation was offered by Suddarth and Herrick (1964) and is equivalent to

$$\text{ASD} = \{S/12\}\{2(1 + P^2)^{1/2} - P^2\ln|\tan[\tfrac{1}{2}\tan^{-1}(P)]| - (1/P)\ln|\tan[\tfrac{1}{2}\tan^{-1}(1/P)]|\} \quad (1)$$

where $P = W/S$ by definition. In what follows, it will be shown that, for any values of S and W, a decrease in ASD can be achieved by geometrically redefining the accepted road and landing configuration of Figure 1.

Consider the effect on ASD of shifting the landings on alternate roads one-half the landing spacing. If the configurations of roads and landings associated with all possible values of

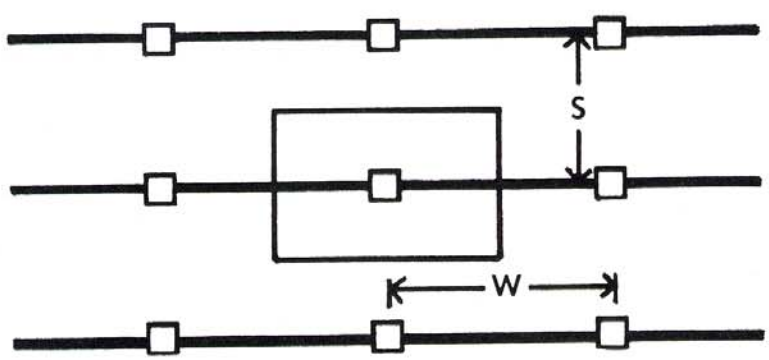

FIGURE 1. The accepted configuration of roads (lines) and landings (squares). S is the road spacing, and W is the landing spacing. Also shown is a representative polygon of the area served by one landing.

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P (i.e., W/S) are plotted, the resultant polygons (Fig. 2) representing the areas served per landing can be grouped into three categories:

- Case 1 hexagons oriented perpendicular to the roads ($P < 2$),
- Case 2 squares with sides oriented at 45° angles from the roads on which the landings are located ($P = 2$), and
- Case 3 hexagons oriented parallel to the roads ($P > 2$).

That shifting does have an effect upon ASD can be easily shown by examining the Case 2 polygon (Fig. 2). If we arbitrarily assume the values $S = 500$ and $W = 1{,}000$, and superimpose the first quadrant of the polygon onto an x-y coordinate system, we obtain the isoceles triangle illustrated in Figure 3, which has been bisected to form two equal right triangles. The length of the common side (a) can be calculated as $500/\sqrt{2}$. As skidding is performed to the origin, the ASD for each of these triangles is equivalent. And, since rotation about the origin in no way affects ASD, we examine only the upper triangle rotated negative 45°.

The general ASD formula for a right triangle as given by Suddarth and Herrick (1964) is

$$\mathrm{ASD} = (1/3)(a^2 + b^2)^{1/2} - (a^2/3b)\ln|\tan[\tfrac{1}{2}\tan^{-1}(a/b)]| \qquad (2)$$

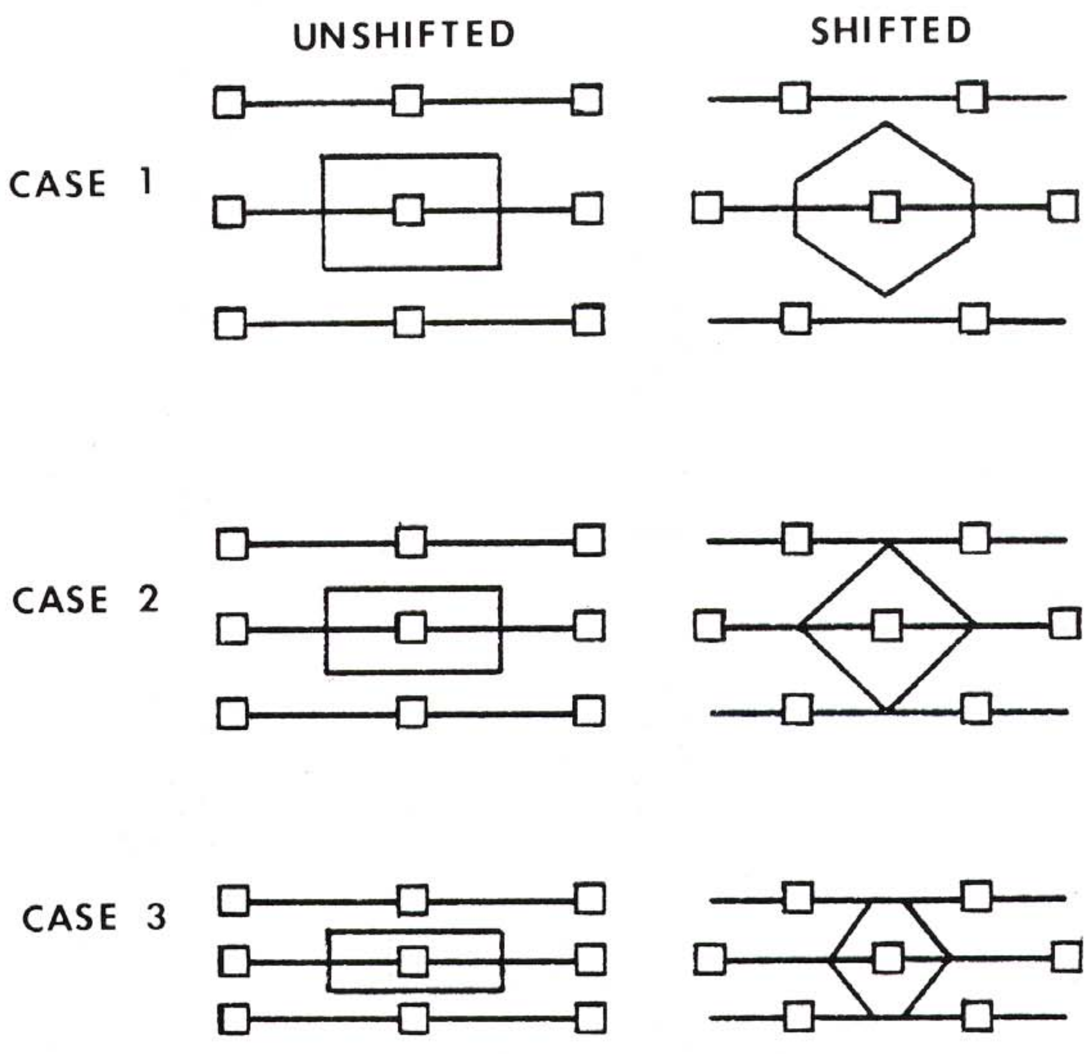

FIGURE 2. The three possible configurations of roads and landings associated with shifting, and their unshifted rectangular counterparts.

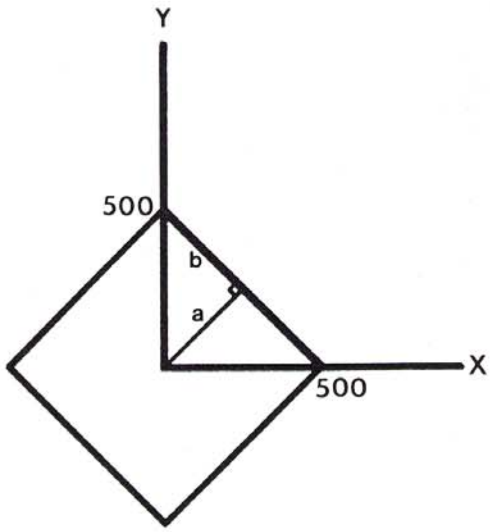

FIGURE 3. The case 2 polygon (square) superimposed onto an x-y coordinate system, where $S = 500$ and W 1,000.

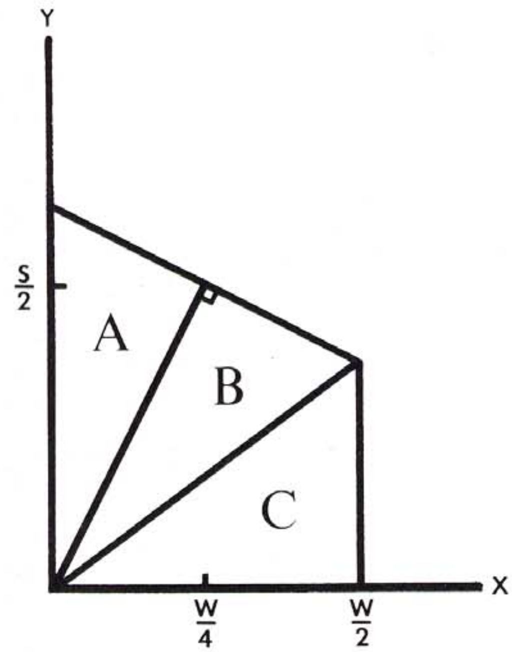

FIGURE 4. The first quadrant of the Case 1 polygon, where skidding is performed to the origin. The area is subdivided into the three right triangles A, B, and C so that Equation 2 can be utilized. Triangles A and B are equal.

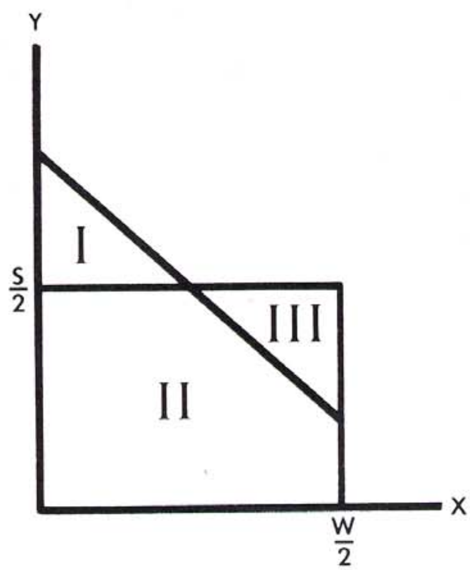

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FIGURE 5. Superimposition of the first quadrant of a polygon associated with shifting and not shifting. The unshifted polygon is made up of II and III, whereas the shifted polygon is comprised of I and II. The areas of I and III are equal.

where a and b are defined in Figure 3. So, to calculate the ASD for the Case 2 shifted polygon, we make the assignments $a = b = 500/\sqrt{2}$ and solve, using Equation 2, which yields ASD = 270.538. The unshifted ASD, calculated by Equation 1, is 296.617. Therefore, for the unique Case 2, ASD can be decreased by 8.79 percent through shifting.

Considering Case 1 and 3, Case 1 situations appear to hold the most real-world promise of being candidates for shifting. Case 3 is suspect, as the practicality of skidding logs which lie near a given road to another road S feet away is questionable. Mathematically, Cases 1 and 3 are quite similar, as only the orientation of the hexagon is changed, however, no presentation of the Case 3 ASD will be offered. For now, an overview of the derivation of the Case 1 ASD formula is given. A more complete version can be found in Bryer (unpublished MS thesis, University of Måine, Orono, Me. 1981).

The formula derivation centers, as above, on superimposing the first quadrant of the Case 1 polygon onto an x-y coordinate system, where the origin is placed at the center of the landing (Fig. 4). By determining the lengths of the sides of the three triangles in Figure 4, we can determine the a and b terms associated with Suddarth and Herrick's Equation

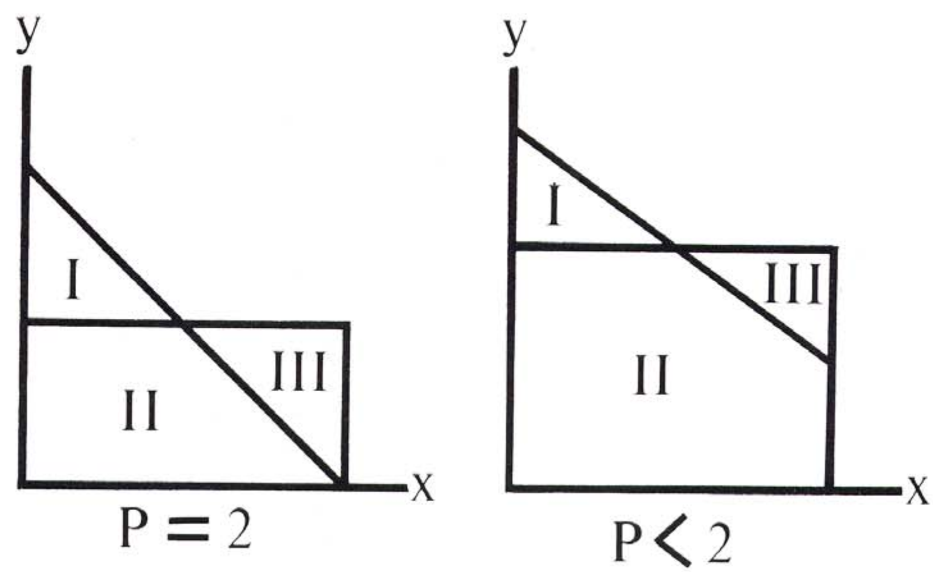

FIGURE 6. The trade-off associated with different values of P.

2. (That information can also be used to show that the area served per landing is unaltered by shifting, and that the maximum skidding distance is decreased by shifting.)

As triangles A and B (Fig. 4) are equal, we need only to calculate the ASD for triangles A (or B) and C and average them, weighted accordingly, to arrive at the Case 1 ASD formula

$$ASD = [(4S^2 + W^2)/(24S)] + [(W^2/12S)\ln|2S/W|] + [(4S^2 + W^2)^{3/2}/(48SW)]\ln|[(4S^2 + W^2)^{1/2} + W]/(2S)|. \quad (3)$$

A FORTRAN program was written to assess the effect of shifting versus not shifting for the Case 1 situation. Equation 1 was used to calculate the unshifted ASD, and Equation 3 was used to calculate the shifted ASD. Examination of the output generated by the

TABLE 1. A comparison of shifting versus not shifting, when P *is less than or equal to 2.*

$P = W/S$	ASD with shift	ASD without shift	Percent decrease achieved through shift
2.00	270.538	296.617	8.792
1.90	277.450	300.512	7.674
1.80	284.718	304.945	6.633
1.70	292.445	310.023	5.670
1.60	300.766	315.884	4.786
1.50	309.853	322.702	3.982
1.40	319.933	330.706	3.258
1.30	331.308	340.200	2.614
1.20	344.389	351.595	2.049
1.10	359.744	365.458	1.564
1.00	378.181	382.598	1.154
.90	400.885	404.193	.818
.80	429.652	432.036	.552
.70	467.339	468.975	.349
.60	518.751	519.804	.202
.50	592.612	593.233	.105
.40	706.492	706.815	.046

program (Table 1) shows a decrease in ASD achieved through shifting, and was calculated as

$$100[(\text{no-shift ASD}) - (\text{shift ASD})]/(\text{no-shift ASD}). \quad (4)$$

Note that when $P = 2$, Equation 3 calculates the same ASD as our detailed treatment above, so Equation 3 can handle the Case 2 situation.

For values tested, the numerical advantage of shifting diminishes as P diminishes, and the two ASD values (shift versus no-shift) converge quite rapidly. To understand the reason for this convergence, consider the superimposition (Fig. 5) of the first quadrants of the polygons resulting from shifting and not shifting, for arbitrary values of S and W. Through shifting, the area in III is lost and the area in I is gained. The contribution toward ASD from the area in II is equivalent whether shifting or not, and hence can be disregarded in this discussion of convergence.

Any difference in ASD values must result from this trade-off from III to I. Consider the cases of $P = 2$ and $P < 2$ in Figure 6. When $P = 2$ the area involved in the trade-off is a much larger percentage of the total area than when $P < 2$, so the magnitude of the difference in ASD values can be expected to be much greater. The result is that, as P becomes smaller, the percentage of the total area unaffected by shifting becomes larger and the ASD values will necessarily converge. It is clear why the greatest difference in ASD values occurs when $P = 2$. There is no configuration in which a larger percentage of the total area participates in the trade-off, and this includes Case 3 situations.

In conclusion, though a mathematical (theoretical) decrease in ASD results from shifting, the practical significance is suspect. More importantly, what effect shifting may have on road and landing spacings and the associated costs has yet to be shown. This would require a reformulation of existing methods of calculating them, as they are based upon the geometry of unshifted roads. However, the magnitude of expenditure necessary for logging road construction is justification enough for further evaluation of this phenomenon.

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